# Quantum & Nuclear Physics

FIZIKA SPhO Training

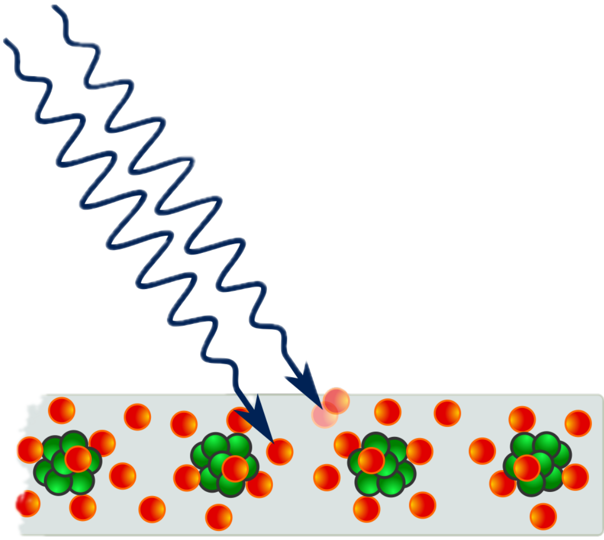
October 2025

## Contents

# 1 Notes

## 1.1 Quantum

### 1.1.1 Photoelectric Effect



The Particle Theory of Light, theorised as a Photon, began with the Photoelectric Effect. It was observed that light behaved not as waves but as packets of energies, each containing,

$$E = hf \tag{1}$$

where $h$ is Plank's constant.

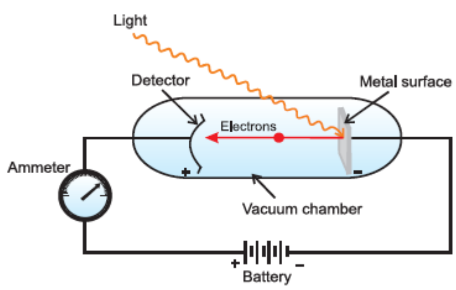
Such Photons can bombard surface electrons and kick them out of the metal sheet. However, only photons with sufficient energy have this ability. The minimum energy required to create a Photoelectron is the Work Function, $\Phi$. The work function is a constant specific to each metal

$$E_{min} = \Phi \tag{2}$$

Finding the Maximum Kinetic Energy of the photoelectron comes down to Conservation of Energy

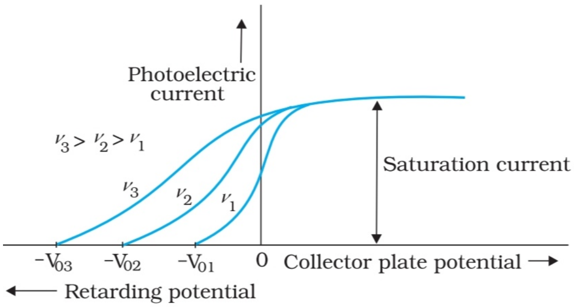
$$KE = hf - \Phi \tag{3}$$

**Example 1.1.** Assuming we shine light of frequency, $f$, on a metal with work function, $\Phi$
(a) What is the minimal stopping potential, $V_s$ of the battery in order to have no reading in the ammeter
(b) Plot a graph of $V_s$ against $f$
(c) Suppose we want to find the $\Phi$ of the metal, how would you experimentally obtain it.



$$KE_{max} = hf - \Phi$$
$$KE = eV_s$$
$$V_s = \frac{h}{e}f - \frac{\Phi}{e}$$

For a given setup, as the potential difference across the plates increases, the photoelectric current[1] increases as such.




Obviously, if the potential difference is more negative than the stopping potential, any photoelectron created at the surface has insufficient energy to reach the detector plate.

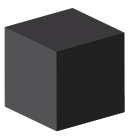
### 1.1.2 Matter Waves

If waves can be particles, can particles be waves? Yes. De Broglie hypothesised that each moving particle has a wave associated to it. The wavelength of this wave is,

$$\lambda = \frac{h}{p} \tag{4}$$

If $p = \frac{E}{c}$, as it should be for light, we obtain back the previous expression, $E = hf$.

**Example 1.2.** Suppose we had a solid black box with mass $m$ at rest on a rough surface with coefficient friction, $\mu$.
(a) If we are tasked to move the box using only a flashlight, what is the minimum power of the flashlight
(b) If the box was covered with a reflective coating, how would your answer to the previous part change



(a)

$$P = \frac{dE}{dt} = hf\frac{dn}{dt}$$
$$F = \mu mg = \frac{dp}{dt} = \frac{hf}{c}\frac{dn}{dt}$$
$$\frac{dn}{dt} = \frac{\mu mg \cdot c}{hf}$$

[1] Current due to the continuous of flow of photoelectrons from metal surface to detector

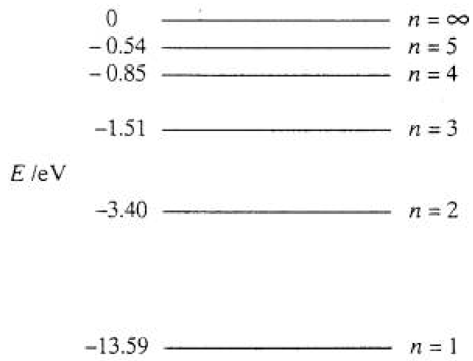
$$P = \mu mg \cdot c$$

(b)

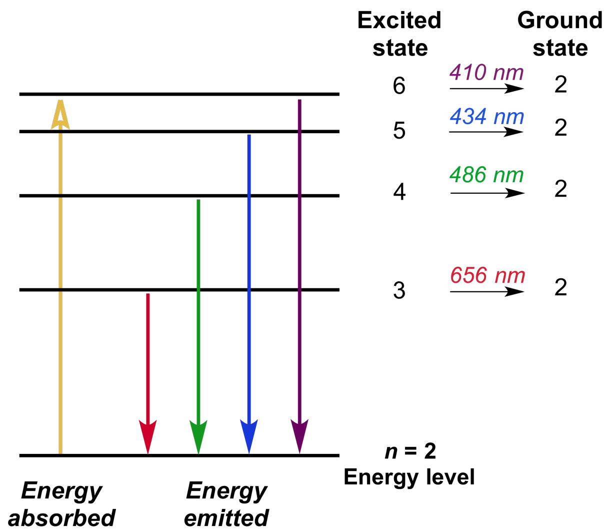
$$P = \frac{1}{2}\mu mg \cdot c$$

### 1.1.3 Excited Atoms

As we learn in Chemistry, electrons are arranged in different shells representing different energy levels. As the shell number increases, the energy of the electron increases.




Using photons, we are able to provide energy and excite low energy electrons up to higher energy shells. This forms an excited state atom which is unstable.




A key thing to note is that, only photons that posses **exactly** the same energy as the energy gap is able to excite the electron. Otherwise, a completely different Compton Effect, occurs.

$$E_\gamma = E_{m \to n} \tag{5}$$

An Excited Atom is unstable and will de-excite to lower energy levels. This energy is emitted as a photon.

$$E_{n \to m} = E_\gamma \tag{6}$$

### 1.1.4 Hydrogen Atom

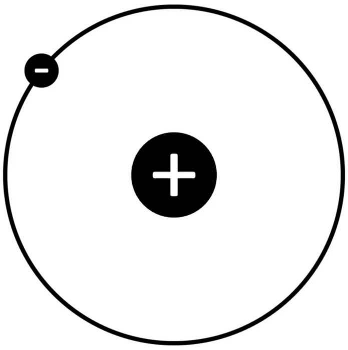
Bohr hypothesised the hydrogen atom as an electron orbiting the proton. The only condition is that the angular momentum of the electron is quantised.

$$mvr = n\hbar \tag{7}$$

Equivalently, one can say that the wavelength of the electron has to be a whole number multiple of the perimeter of orbit and arrive at the same result.

$$n \times \lambda = 2\pi r \tag{8}$$

Electrostatic force between the proton and electron acts as a central force and the electron undergoes uniform circular motion.



$$\frac{e^2}{4\pi\epsilon_0 r^2} = m\frac{v^2}{r} \tag{9}$$

The following two statements can be used to find the energies of the different energy levels and their corresponding orbital radius.

$$v^2 = \frac{n^2\hbar^2}{m^2r^2}$$

$$r = \frac{4\pi\epsilon_0\hbar^2}{e^2m} \times n^2 \tag{10}$$

$$E = KE + PE = \frac{e^2}{8\pi\epsilon_0 r} - \frac{e^2}{4\pi\epsilon_0 r} = -\frac{e^2}{8\pi\epsilon_0 r}$$

$$E = \frac{e^4m}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2} = 13.59\frac{1}{n^2}\text{ eV} \tag{11}$$

**Example 1.3.** Instead of a hydrogen atom, lets take a nucleus with $Z$ protons and a single orbiting electron. Find the energy levels and each corresponding orbital radius

The previous two condition still hold. Just that the electrostatic force is now stronger.

$$mvr = n\hbar$$

$$\frac{Ze^2}{4\pi\epsilon_0 r^2} = m\frac{v^2}{r}$$

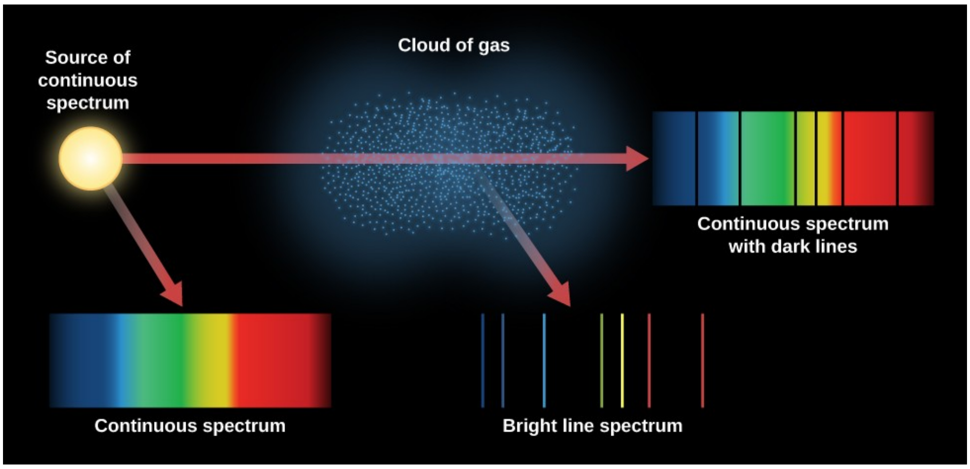
$$r = \frac{4\pi\epsilon_0\hbar^2}{Ze^2m} \times n^2$$

$$E = -\frac{Ze^2}{8\pi\epsilon_0 r}$$

$$E = -\frac{Z^2e^4m}{32\pi^2\epsilon_0^2\hbar^2}\frac{1}{n^2}$$

### 1.1.5 Spectra

Light can be shined at a diffraction grating to see the spread of all the various colors in the light.



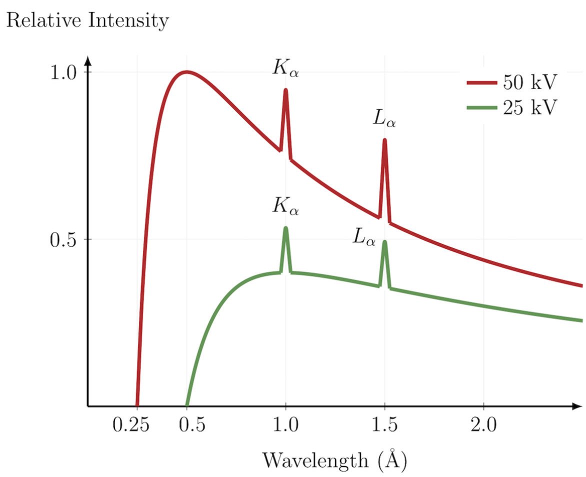

**Continuous Spectrum** arises when there is no material medium in between the source and observer.

**Emission Spectrum** arises when there is no direct source, but a cloud of gas that is constantly excited and de-excited. Only light corresponding to the possible de-excitations will be seen in the emission spectrum.

**Absorption Spectrum** arises when there is a strong direct source but is blocked by a cloud of gas. It will absorb light corresponding to the possible excitations.

**Remark.** Technically, the absorbed light frequencies can be emitted when the gas de-excites. However, the intensity of this is negligible relative to the rest of the light from the direct source

When high speed electrons are collided with a target material, the X-Ray spectrum forms.




The X-Ray spectrum is the superposition of two distinct spectrum.

1. The Continuous Spectrum (**Bremsstrahlung**)

When electrons decelerate, they release photons to conserve energy. As the extent at which the electrons slow down varies, there is a continuous spectrum of light.

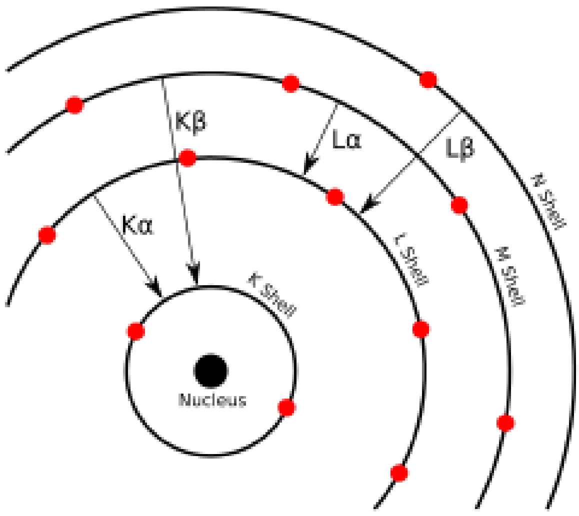
You should be able to find the minimum wavelength, $\lambda_0$, or the maximum frequency, $f_0$. This occurs when all the electron's energy is converted into a single photon.

$$hf_0 = KE_e \tag{12}$$

As such, $\lambda_0$ is dependent on the initial Kinetic Energy of the electron and not the material.

2. The **Characteristic X-Rays**

While the underlying principle is the same, the mechanism is a little different than the previous line spectras.




Instead of exciting an electron to a higher energy level, we eject the electron out of the atom completely. This results in a hole in the lower energy shell. A hole that will be rapidly filled by high energy electrons. In this process, a photon is released corresponding to the characteristic peaks.

$$hf = E_{n\to m} \tag{13}$$

An annoying thing is that you would have to familiarise yourselves with the weird convention.

- Shell 1 $\to$ K, Shell 2 $\to$ L, Shell 3 $\to$ M...
- 1 Shell jump $\to \alpha$, 2 Shell jump $\to \beta$...

As you may notice, this peaks are characteristic to the material and are independent to the initial Kinetic Energy of the electron

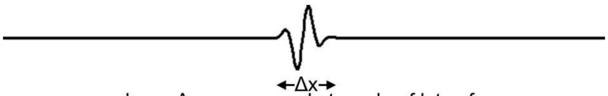
## 1.2 Heisenberg Uncertainty

The Heisenberg Uncertainty principle states that one cannot measure both the position and momentum of a particle to absolute precision.

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{14}$$

Using the De Broglie wavelength, we can relate the momentum of a particle to its wavelength

- If we want a lower uncertainty in the position of the particle, we have greater uncertainty in the wavelength of the particle




- If we want a lower uncertainty in the wavelength of the particle, we have greater uncertainty in the position of the particle


## 1.3 Radioactive Decay

### 1.3.1 Decay Law

This would be the most commonly tested portion of Nuclear Physics in SPhO due to its wider range of testable questions that are in line with the IPHO syllabus. Regardless, most of these questions can be solved using the Decay Law which you will learn at the end of JC2.

$$\frac{dN}{dt} = -\lambda N \tag{15}$$

The Decay Law states that the rate of decay, $-\frac{dN}{dt}$ is directly proportional to the remaining number of nuclei, $N$, with proportionality constant $\lambda$. This constant is commonly referred to as the decay constant.

### 1.3.2 Motivation

I will attempt to motivate the Decay law so that you can apply similar skills elsewhere and also appreciate the underlying principle.

At the heart of it lies,

$$dP = \lambda dt \tag{16}$$

The probability that a nuclei is going to decay in the next $dt$ interval is a constant $dP$[2]. It is nice to imagine the nuclei rolling a dice at every $dt$ interval to decide its own fate.

This fact is profound and can help us interpret the Decay Law. Suppose that we have a singular unstable nuclei with probability of decay defined as[3]

$$dP_{decay} = \alpha dt \tag{17}$$

The probability that the nuclei does **not** decay in the next $dt$ interval is,

$$dP_{survive} = 1 - \alpha dt \tag{18}$$

To find the probability that the nuclei survives a time period $t$, we can split $t$ into an infinite number of $dt$ intervals.

$$dt = \lim_{n\to\infty} \frac{t}{n} \tag{19}$$

[2]Where this comes from is far beyond the syllabus. If interested, do more quantum

[3]I will use $\alpha$ as we have yet to prove that $\alpha = \lambda$

Subsequently, for the nuclei to not decay throughout $t$, it has to survive each $dt$ interval with its corresponding probability. Since these are independent events,

$$P = (dP_{survive})^n \tag{20}$$

$$P = (1 - \alpha dt)^n = \lim_{n \to \infty} \left(1 - \alpha \frac{t}{n}\right)^n \tag{21}$$

From the definition of the natural number $e$,

$$P = e^{-\alpha t} \tag{22}$$

We have now established the probability for a single nuclei to survive $t$. For a large group of identical nuclei we can approximate the number of nuclei that survive over a period $t$ using a binomial distribution[4] with $N_0$ trials and $P = e^{-\alpha t}$. The mean of a binomial distribution is $\overline{X} = N_0 \cdot P$. Therefore, the expected number of particles that survive after $t$ is,

$$N = N_0 e^{-\alpha t} \tag{23}$$

Finally, we can get back the form that we are all more familiar with, noting that the probability for decay per unit time is numerically equal to the decay constant, $\alpha = \lambda$

$$\frac{dN}{dt} = -\lambda N \tag{24}$$

#### 1.3.3 More Probability

To find the probability that a nuclei will decay at some specific time, $t$, we need to find the probability that it first survives until time $t$ and the probability of it subsequently decaying in the next $dt$ interval.

$$dP(t) = P(\text{Survive till } t) \times P(\text{Decay in next } dt) \tag{25}$$

$$dP(t) = \lambda e^{-\lambda t} dt \tag{26}$$

This $dP(t)$ is the Probability Density Function which attaches a probability to the chances of decay at each $t$ to $t + dt$ interval. To reiterate, the probability of decay in each $dt$ interval is still constant. However, since the nuclei has to also survive till $t$ for it to decay at $t$, the probability decreases with increasing $t$.

**Example 1.4.** Find the mean life time of a nuclei with decay constant $\lambda$.

The mean life time of the nuclei is in other words the expectation value for $t$.

$$\tau = \int_0^\infty t \cdot dP(t) = \int_0^\infty t \lambda e^{-\lambda t} dt$$

$$\tau = \frac{1}{\lambda}$$

[4] This is in the H2 Math syllabus. If unfamiliar search up Discrete Random Variables

### 1.3.4 Half-Life

Most of the time, we will get the half-life of the nuclei instead of its decay constant. The half-life of a nuclei is defined as the time taken for the undecayed nuclei population to halved.

$$T_{1/2} = \frac{\ln 2}{\lambda} \tag{27}$$

Do not jump to calculate the decay constant when given half-lives. Half-life data can be intuitive when considering decay. Essentially, we determine the number of half-lives that has passed and times that many $\frac{1}{2}$ to the initial nuclei population, $N_0$.

$$N = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}} \tag{28}$$

**Example 1.5.** Find the half-life of a nuclei with mean life time $\tau = \frac{10}{\ln 2}$ s and hence find the number of nuclei left after 30 s given that the original population was 1000.

By definition,

$$\frac{1}{2}N_0 = N_0 e^{-\lambda T_{1/2}}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2 = 10 \text{ s}$$

| t/s | No. of Half-lives | Undecayed Nuclei |
|---|---|---|
| 0 | 0 | 1000 |
| 10 | 1 | 500 |
| 20 | 2 | 250 |
| 30 | 3 | 125 |

### 1.3.5 Decay ODE

When dealing with multiple sources and sinks that can produce or reduce the nuclei population we need to modify the Decay Equation. To find the net rate of change in the population, we can sum up all these terms linearly.

$$\Delta N = \sum(\text{Rate of Production}) \times \Delta t + \sum(\text{Rate of Removal}) \times \Delta t \tag{29}$$

As $\Delta t \to 0$,

$$\frac{dN}{dt} = \sum Q_i \tag{30}$$

A useful mathematical tool to find N(t) given complicated source and sink terms is the Integrating Factor. I have provided a few questions for you to practice using the integrating factor method.

**Example 1.6.** Lets do a classic problem to test our understanding. Suppose we only have $N_0$ parent nuclei at $t = 0$ and no daughter nuclei. The parent has a decay constant of $\lambda_1$ and decays into an unstable daughter nuclide with decay constant $\lambda_2$. Find the number of undecayed daughter nuclei as a function of $t$.

As mentioned previously, lets identify each source and sink term. Of course, the unstable daughter nuclide will decay at $\lambda_2 N_2$. Additionally, since every parent nuclei that decays becomes a singular daughter nuclide, the rate of production of daughter nuclei is equal to the decay rate of the parent nuclei, $\lambda_1 N_1$.

$$\frac{dN_2}{dt} = -\lambda_2 N_2 + \lambda_1 N_1$$

Do note that $N_1$ is itself also decaying hence,

$$N_1 = N_0 e^{\lambda_1 t}$$

In this case, the appropriate integrating factor is $e^{\lambda_2 t}$

$$\frac{dN_2}{dt} e^{\lambda_2 t} + \lambda_2 N_2 e^{\lambda_2 t} = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t}$$

$$\frac{d}{dt}(N_2 e^{\lambda_2 t}) = \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t}$$

$$\int_0^{N_2 e^{\lambda_2 t}} d(N_2 e^{\lambda_2 t}) = \int_0^t \lambda_1 N_0 e^{(\lambda_2 - \lambda_1)t} dt$$

$$N_2 e^{\lambda_2 t} = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 (e^{(\lambda_2 - \lambda_1)t} - 1)$$

$$N_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} N_0 (e^{-\lambda_1 t} - e^{-\lambda_2 t})$$

### 1.3.6 Secular Equilibrium

A commonly tested concept to simplify complex decay chains is secular equilibrium. This phenomenon occurs when the half-life of the parent nuclei is significantly greater than its daughter nuclide.

$$\lambda_2 >> \lambda_1,\ \lambda_3 >> \lambda_1,\ ... \tag{31}$$

In this special case, the number of daughter nuclei will reach such an equilibrium state, where the decay rate of the parent nuclei is equal to the decay rate of the daughter nuclei.

$$N_i \lambda_i = N_1 \lambda_1 \tag{32}$$

**Example 1.7.** Prove that $N_2 \lambda_2 = N_1 \lambda_1$ in the context of Example 1.3 if secular equilibrium conditions are met, $\lambda_2 >> \lambda_1$

## 1.4 Nuclear Reactions

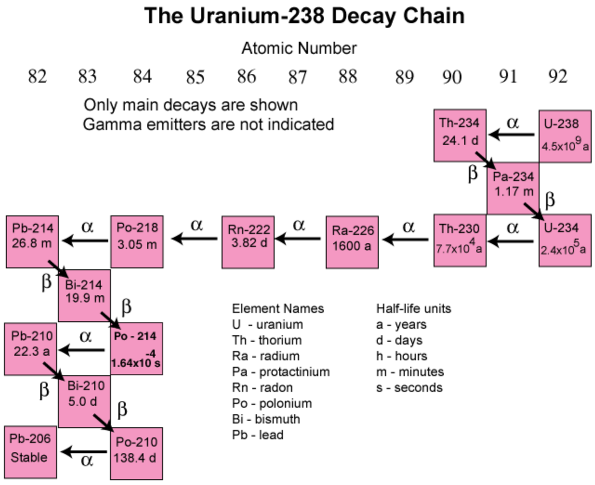
### 1.4.1 Mass Defect

Everyone has heard of $E = mc^2$. This is when you apply this relationship. The more massive an object, the greater energy it posses. Likewise, if an object emits energy, it loses mass. Proof of this principle was observed in the mass defect of the nuclei. Alone, 2 Protons and 2 Neutrons have a theoretical combined mass of,

$$M' = 2m_p + 2m_n = 4.03188u$$

Yet, when combined to form a $^4He$, $M = 4.00260u$

$$\Delta m = M' - M = 0.02928u$$

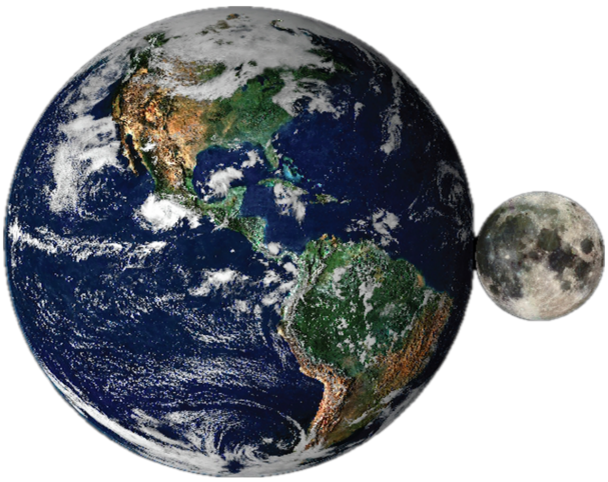



The mass defect and total mass of the nuclei is,

$$\Delta m = \frac{E_b}{c^2} \tag{33}$$

$$M = Zm_p + Nm_n - \Delta m \tag{34}$$

**Example 1.8.** Suppose the moon was abruptly shifted from its original position to right next to Earth. Determine how much *mass* was lost in the earth-moon system.



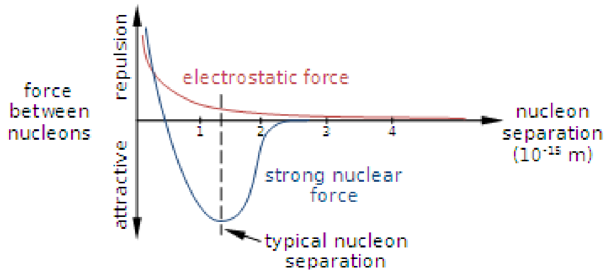
There is a loss of gravitational potential energy when the moon is closer to earth. This results in a decrease in the mass of the earth-moon system.

$$\Delta m = \frac{\Delta U_g}{c^2} = \frac{GMm}{c^2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

### 1.4.2 Strong Nuclear Force

Strong Nuclear Force is the force that bind the nucleons in the nucleus. It is significantly attractive when nucleons are close to one another and repulsive when too close.
Similar to gravity in the above example, energy is lost in the system when nucleons approach each other. By conserving energy, the total decrease in potential energy during the formation of the nuclei is the energy emitted known as the Binding Energy, $E_b$.

$$E_b = \sum \Delta U \tag{35}$$

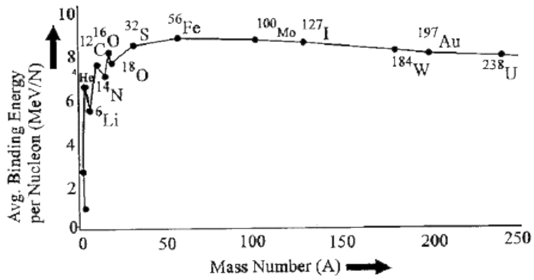



The binding energy is different for each nuclide and depends on the number of neutrons and protons. This is why some nuclei are considered more stable than other. The Semi Empirical Mass Formula (SEMF) gives us a crude estimate of $E_b$.

$$E_b = a_v A^{1/3} - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + \delta \tag{36}$$

**Remark.** For SPhO, binding energies or mass defects would be most definitely given if needed. Deeper understanding of strong nuclear force is also not required.

**Example 1.9.** The following is a graph of Binding Energy per Nucleon, $\frac{E_b}{A}$ over Atomic Number, $A$. Convince yourself why Fe is relatively more stable.




### 1.4.3 Q-Value

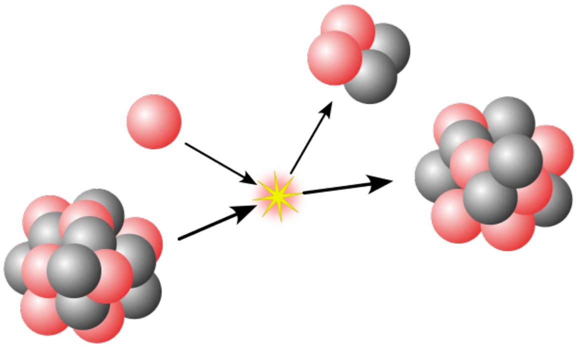
The Q-Value of a nuclear reaction is defined as the energy released due to the decrease in mass of the system. If the mass of the system increases, Q-Value is negative.

$$Q = \Delta M c^2 \tag{37}$$

**Remark.** Do note that the $\Delta M$ here is not the mass defect of a single nuclide but the net change in the mass of the system before and after the reaction.

Finding Q-Value is relatively easy in most cases. However, remembering to use the Q-Value when conserving energy during nuclear reactions is the real challenge.

$$\sum KE_f = \sum KE_i + Q \tag{38}$$



**Example 1.10.** Consider the following reaction, ${}^{14}N(n,p){}^{14}C$. Find the Q-Value of this reaction in terms of the masses of the relevant nuclides. Hence or otherwise, find the Q-Value in terms of the Binding energies of the relevant nuclides.

$$Q = (m_N + m_n - m_C - m_p)c^2$$
$$Q = (7m_n + 7m_p - \Delta m_N + m_n - (8m_n + 6m_p - \Delta m_C) - m_p)c^2$$
$$Q = (\Delta m_C - \Delta m_N)c^2 = E_C - E_N$$

An alternative way to interpret the final expression is to imagine you needing to pump $E_N$ into the system to break the Nitrogen Nuclei and as a result releasing $E_C$ from the formation of the Carbon Nuclei.

### 1.4.4 Collisions

In the subatomic world, there exists no inelastic collisions. All collisions conserve energy and momentum.

$$\sum KE_f = \sum KE_i + Q \tag{39}$$

$$\sum \vec{p}_f = \sum \vec{p}_i \tag{40}$$

In non-relativistic speeds, Kinetic Energy can be written as,

$$KE = \frac{p^2}{2m} \tag{41}$$

Therefore, Energy Conservation can be rewritten in terms of momentum, $p$. This gives 2 simultaneous equations that can be solved for the unknowns.

$$\sum \frac{p_f^2}{2m} = \sum \frac{p_i^2}{2m} + Q \tag{42}$$

$$\sum \vec{p}_f = \sum \vec{p}_i \tag{43}$$

At relativistic speeds, the approach is similar. Just that a better way to relate momentum and energy is required. Using the Lorentz invariance helps,

$$E^2 = p^2c^2 + m_0^2c^4 \tag{44}$$

Rewriting the Momentum Conservation, we will once again attain 2 simultaneous equations,

$$\sum \sqrt{E_f^2 - m_0^2c^4} = \sum \sqrt{E_i^2 - m_0^2c^4} \tag{45}$$

$$\sum E_f = \sum E_i + Q \tag{46}$$

With sufficient information and limited particles, we can solve for possible Kinetic Energies that the product particles can have.

### 1.4.5 Lorentz Transformation

In certain cases, there is a need to transform from one frame to another which would alter the momentum and energies of the particles. To transform their momentum and energies use the Lorentz Matrix,

$$\begin{pmatrix} E'/c \\ p' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} E/c \\ p \end{pmatrix} \tag{47}$$

**Example 1.11.** Suppose a stationary excited state nuclei with excitation energy $E_{exc}$ de-excites, releasing a photon.
(a) Determine the frequency of the photon
(b) What if the nuclei was already moving at relativistic speed $v$ w.r.t. to an observer. What is the observed frequency of the photon. Assume that the photon was emitted in the direction of motion

(a) Do not be too quick to think that the energy of the photon, $E\gamma$ is $E_{exc}$. Some of the $E_{exc}$ will be used for the recoil energy of the ground state nuclei. Conserving momentum and energy,

$$E_{exc} = E_\gamma + E_{rec}$$

$$p_\gamma = p_{rec}$$

Using the Relativistic Dispersion Relation to rewrite the momentum conservation,

$$E_\gamma^2 = E_{rec}^2 - m_0^2c^4$$

$$E_\gamma^2 = (E_{exc} - E_\gamma)^2 - m_0^2c^4$$

$$2E_\gamma E_{exc} = E_{exc}^2 - m_0^2c^4$$

$$E_\gamma = \frac{E_{exc}^2 - m_0^2c^4}{2E_{exc}}$$

$$f = \frac{E_{exc}^2 - m_0^2c^4}{2hE_{exc}}$$

(b) Using the Lorentz Boost,

$$E' = \gamma E - \gamma\beta E$$

$$E' = E\sqrt{\frac{1+\beta}{1-\beta}}$$

$$f' = f\sqrt{\frac{1+v/c}{1-v/c}}$$

# 2 Problems

*Problems are arranged in roughly increasing difficulty.*

**Problem 2.1.** 90% of the potassium-40 decays to form calcium-40 while the remaining 10% decays to form the stable isotope argon-40. In a particular sample of rock, the ratio of the number of potassium atoms to the number of argon atoms is found to be 2:1. Estimate the age of the rock. Assume that originally there was no argon present.

**Problem 2.2.** Suppose we have a $Li^{2+}$ cation. a) what are the energy levels that the electron can possess. b) If light of frequency $2.947141 \times 10^{16}$Hz constantly excites a ground state $Li^{2+}$ ion, how many possible emission lines would be observed.

**Problem 2.3.** A parent isotope has a half-life of $10^4$ years. It decays through a series of radioactive daughters to a final stable isotope. Among the daughters the greatest half-life is 20 years. Others are less than a year. At $t = 0$, one has $10^{20}$ parent nuclei but no daughters. (i) At $t = 0$ what is the activity (decays/sec) of the parent isotope? (ii) How long does it take for the population of the 20 yr isotope to reach approximately 97% of its equilibrium value?

**Problem 2.4.** 14C decays with a half-life of about 5500 years. (i) What would you guess to be the nature of the decay, and what are the final products? Very briefly explain. (ii) If no more 14C enters biological systems after their death, estimate the age of the remains of a tree whose radioactivity (decays/sec) of the type given in (i) is $\frac{1}{3}$ of that of a comparable but relatively young tree.

**Problem 2.5.** Suppose a radiopharmaceutical drug undergoes alpha decay with decay constant, $\lambda_a$. At the same time, the body randomly and spontaneously secretes the drug from the body with a biological decay constant of $\lambda_b$. Given that the drug undergoes alpha decay, what is the probability it decays inside the body?

**Problem 2.6.**

Suppose you had a box of $N$ radioactive particles of species $N_1$, which follow the following bizarre decay chain. Describe quantitatively the distribution of radioactive particle species present in the box after a long time. How much time must elapse before the probability that no $N_1$ remains is more likely than not.

$$N_1 \xrightarrow{\lambda} N_2 \xrightarrow{\lambda} N_3 \xrightarrow{\lambda} N_4 \xrightarrow{\lambda} \ldots$$

**Problem 2.7.** Consider when we have a "square wave" source which alternates between being active with source strength $S_0$ for a duration of $T$, and being inactive for a duration of T. For such a time-varying source $S(t)$, the resulting nuclide population $N(t)$ is also time-varying. However, as we can see from Figure 1, the nuclide population $N(t)$ eventually approaches a steady-state behaviour. (i) Derive an expression for the steady-state peak population (Figure 1 in black-dashed line) (ii) Does the initial condition $N(0)$ affect the steady-state behaviour? Prove analytically. (iii) However you derived your expression in part (i), there is another approach through which you could have arrived at the same answer. Find it.

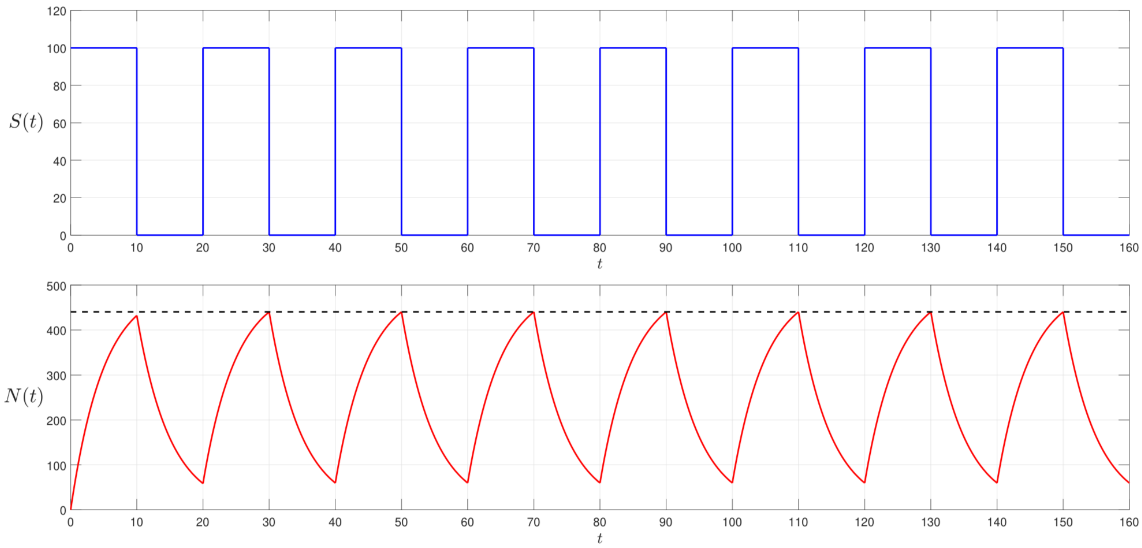



Figure 1: Top: Square wave source (in blue solid line). Bottom: Time-varying nuclide population (in red solid line) with steady-state peak population (in black-dashed line).
Parameters used: $S_0 = 100$, $\lambda = 0.2$, $T = 10$, $N(0) = 0$.